

PHYS 345 Laboratory: Charge to Mass Ratio of the Electron

Purpose: To determine the charge to mass ratio of the electron and compare it with the accepted value.

Apparatus: CENCO e/m apparatus (electron tube with Helmholtz coils)
High-voltage DC power supplies (2)
High-current power supply
0 – 6 VAC power supply
Current reversing switch
Multimeters (4), ruler, digital caliper, meter stick with caliper jaws
Compass, flashlight

References

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Instruction Manual, Specific Charge of Electron Apparatus and Electron Beam Tube, Central Scientific Company.
J.J. Thomson, Phil. Mag. **44**, 295 (1897).
J.R. Reitz and F.J. Milford, Foundations of Electromagnetic Theory, Addison Wesley (1969), p. 156-57.
K.T. Bainbridge, The American Physics Teacher **6**, 3 (1938).
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Introduction

Prior to Michael Faraday's work on electrochemical processes in 1839, electricity was considered a "fluid" that could be added or subtracted in a continuous fashion from objects. It was subsequently discovered that metals emitted negative electrical current when heated, illuminated by light, or subjected to a strong electric field. There was evidence that the negative current was comprised of particles carrying a negative charge, but several scientists believed in wave phenomena and hence the negative currents were called cathode rays. These cathode rays were found to be detected universally whenever a negative current was emitted from an electrode. Surprisingly, they were not related to the particular metal from which the emitting electrode was fabricated, thus providing strong evidence for their fundamental nature.

J.J. Thomson believed that cathode rays were particles, and was able to prove in

1897 that the charged particles emitted from a heated electrical cathode were in fact the same as cathode rays. Thomson's experiments combined both ingenious insight and the use of recent advances in vacuum pumps and charge measuring devices (electrometers). Using clever calorimetric techniques, he measured the temperature rise of thin metal targets inserted into a glow discharge (first produced by Faraday, when two metal plates were inserted inside an evacuated glass tube and raised to a high electrostatic potential). This strategy enabled Thomson to calculate the energy imparted to the metal targets by the invisible particles responsible for producing the glow. By cleverly assuming the invisible particles followed Newton's laws of motion, he was able to estimate the velocity of the particles responsible for the glow discharge. He also used crossed electric and magnetic fields to further study the motion of these invisible particles.

As a result, Thomson concluded (within the precision of his measurements) that a negatively charged particle with a *mass to charge ratio* of $(1.3 \pm 0.2) \times 10^{-11}$ kg/C was responsible for producing the glow discharge. He thus proved convincingly that the cathode rays acted as negatively charged particles, called electrons, and received a Nobel Prize in 1906 in recognition of this discovery. The modern, accepted value for the *charge to mass ratio* of an electron is $(1.7588196 \pm 0.0000005) \times 10^{11}$ C/kg.

In this experiment, you will perform a modification of Thomson's original work. By measuring the deflection that a magnetic field produces on a beam of electrons having a known energy, you will deduce a value for the charge to mass ratio of electrons.

Theory

The magnetic force \vec{F} acting on a charged particle of charge q moving with velocity \vec{v} in a magnetic field \vec{B} is given by the Lorentz force law:

$$\vec{F} = q\vec{v} \times \vec{B} \quad . \quad (1)$$

If the charged particle velocity is perpendicular to \vec{B} , then Eq. (1) can be written in scalar form as $F = qvB$. It also implies that the particle trajectory is circular, and from Newton's Second Law the particle must be experiencing a centripetal force of magnitude

$$F = \frac{mv^2}{r} \quad , \quad (2)$$

where m is the particle mass, and r is the radius of the circular motion. Since the only force acting on the particles is that caused by the magnetic field, the magnetic force must equal the centripetal force, so $qvB = mv^2 / r$, or

$$\frac{q}{m} = \frac{v}{Br} \quad . \quad (3)$$

It is somewhat difficult to measure v experimentally, but it can be eliminated from Eq. (3) using conservation of energy considerations. If a charged particle is initially at rest and is accelerated through an electric potential difference V , then the kinetic energy after acceleration is equal to the change of the potential energy qV , or

$$\frac{1}{2}mv^2 = qV \quad . \quad (4)$$

The velocity of the charged particle is therefore

$$v = \sqrt{\frac{2qV}{m}} \quad . \quad (5)$$

From the Biot–Savart law, the magnetic field produced near the axis of a pair of Helmholtz coils (see next section) has a magnitude given by

$$B = \frac{8\mu_0 NI}{R\sqrt{125}} \quad , \quad (6)$$

where N is the number of turns in each Helmholtz coil, I is the current through the Helmholtz coils, R is the radius of the Helmholtz coils, and μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ T·m/A). A derivation of this formula can be found in most introductory texts on electricity and magnetism. Equations (5) and (6) can now be substituted into Eq. (3) to achieve

$$\frac{q}{m} = \frac{e}{m} = \frac{2V}{B^2 r^2} \quad . \quad (7)$$

Since the charged particle in this experiment is the electron, the special symbol e has been introduced which represents its charge.

Experimental Apparatus and Techniques

Since it is difficult to detect a single electron, you will use a beam of electrons which all have approximately the same kinetic energy. In this experiment, the beam is produced by an electron gun mounted in a tube as shown in Fig. 1. A filament heater warms a metal plate, called the cathode, releasing electrons. The emitted electrons pass through a grid, charged to a positive potential with respect to the cathode, which serves to focus the beam. A disk (typically called the “anode”) held at a high positive potential relative to the cathode accelerates the electrons. The accelerated beam is then allowed to pass through a small hole in the disk, causing it to emerge into the domed region of the tube (see Fig. 1). The disk itself is etched with concentric rings of radii 0.50, 1.00, 1.50,

and 2.00 cm which fluoresce (along with the dome) when struck by electrons. The entire electron tube is evacuated of air and backfilled with a trace amount of an inert gas (probably argon) that causes the beam to make a visible trace.

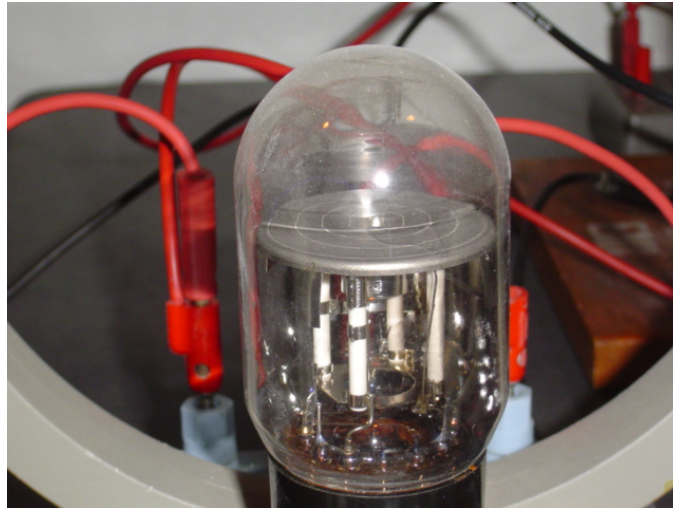


Figure 1. The CENCO electron tube.

A pair of Helmholtz coils, wound on non-magnetic aluminum rims, is used to produce a magnetic field perpendicular to the electron beam. Helmholtz coils take advantage of the fact that the magnetic field along a line perpendicular to the plane of a circular coil of radius R along the coil's central axis varies approximately linearly with distance at a point located at $R/2$ from the center. Therefore, by placing two coils around the same axis, separated by R , the field at the center of the assembly is approximately uniform. With an applied current of about 4 A, the coils produce a magnetic field of about 4×10^{-3} T at the center. Each pair of coils has the number of turns N indicated by the label at its base.

The strength of the magnetic field can be adjusted by changing the current in the coils. Variation of either the accelerating potential in the tube or the strength of the magnetic field causes the radius of the circle described by the electron beam to vary. If the beam follows a semicircular path above the disk and, on returning to the disk, strikes one of the four circles etched on its face, the diameter of the beam's trajectory is equal to the radius of the circle etched on the disk.

The magnetic field needed to produce the observed curved paths for the electron beam is small, so in principle the effect of the Earth's magnetic field must be taken into account. One way to minimize any effects due to the Earth's field is to rotate the apparatus so the local Earth's field is parallel to the motion of the electron beam [see Eq. (1)]. For precise work, the tube and coils should be rotated in order to achieve the proper orientation. The importance of the Earth's field can be qualitatively assessed by observing whether the emerging electron beam is bent slightly when the current in the Helmholtz coils is set to zero. By rotating the e/m apparatus on the table, the orientation which produces a minimum deflection can usually be found. You can use a compass and

the data on page 8 to help you with this. You should also check that there are no other “stray” magnetic fields in the vicinity of the apparatus due to the power supplies, meters, or other sources.

Experimental Procedure

Perform the appropriate wiring to the apparatus as shown in Figs. 2 and 3. Before applying power, have your instructor check your circuit. Remember to orient the apparatus so that the effect of Earth’s magnetic field is minimized.

Energize the tube circuits, and adjust the tube filament voltage to 6 VAC, ensuring that the filament current is about 0.6 A or less (as indicated with an ammeter). Let the filament warm up for about 2 minutes, then apply both plate and grid voltage such that the plate (grid) voltage is in the range of about 80 – 100 V (40 – 60 V), in order to focus the beam into a tight spot on the domed surface of the tube. It will likely be necessary to dim the room lights in order to see the beam.

Before applying current to the coils, note whether the beam appears deflected from a straight-line path. Such deflection would indicate that stray fields are influencing the beam. See if you can minimize any deflection by adjusting the position of the apparatus. Be careful not to touch any high-voltage connections!

Energize the circuit to the coils, and bring the current up slowly. You should see the beam bend and then begin striking the top disk. Adjust the field current and plate potential until the beam strikes one of the etched circles (each circle will fluoresce when struck by the beam). You may need to adjust the grid potential as well in order to keep the beam focused. Use the reversing switch to reverse the direction of the current in the coils and thus the direction of the magnetic field (you can “see” the effect this has using a compass). The beam should flip over, and hit the same marked circle on the opposite side of the disk (you may have to make some minor adjustments in order to achieve the best beam spot). Determine the direction of the magnetic field where the beam is.

After you have become comfortable with the apparatus, make five measurements of different coil currents and plate potentials that make the beam hit the desired spot for each circle and each field direction. Think about the best way to decide when the beam most accurately strikes each circle. In the end, you should have a data set with (at least) 40 points. Estimate the uncertainty in r by eye.

Further suggestions and precautions:

Use proper leads: the banana plug leads and associated spade lugs. (Don’t use the small wire used in electronics class.)

The panel meters on the power supplies are not sufficiently accurate for good data. Use digital meters.

The coil currents should be limited to about 4.5 A. Higher currents than this can cause dangerous heating. (You will hear a crackling sound as the coils warm — some of this is normal.)

Monitor the grid current with an ammeter (it should be about 0.6 A or less) and

the plate current using the corresponding power supply (it should be no more than a few milliamps). If these currents get too large, turn the filament voltage down and/or adjust the grid potential. Excessive grid current will heat and damage the potentiometer and reduce tube life.

With long leads that carry the same current, you can minimize the magnetic field they produce by twisting them together. However, try to minimize the use of long wires in favor of smaller ones when possible.

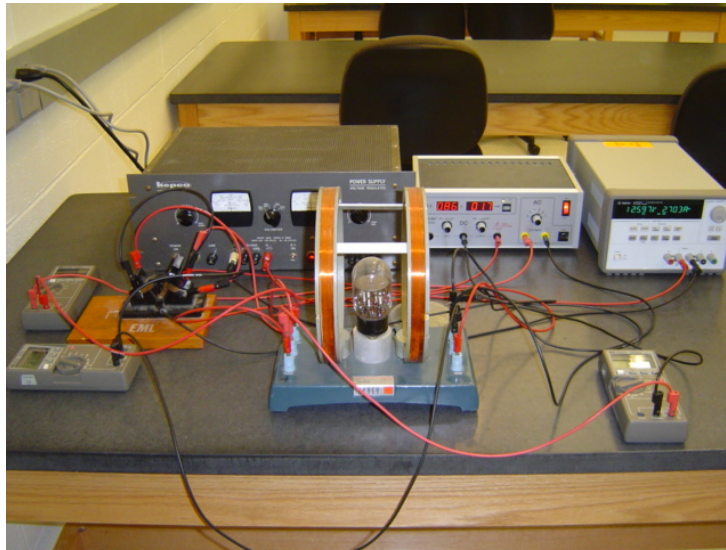


Figure 2. Setup for the e/m experiment.

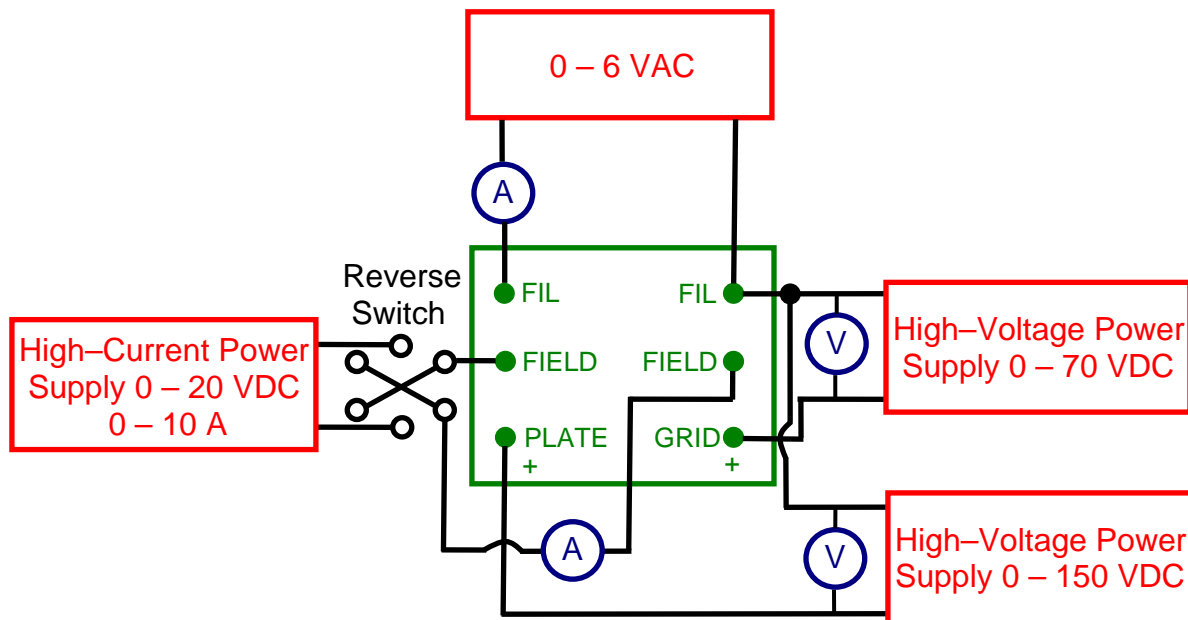


Figure 3. Circuit diagram for the e/m experiment.

Data Analysis and Discussion

Measure the Helmholtz coil geometry: the coil width w , inner and outer radii R_a and R_b , and coil separation s , measured from the center of the coil. From these measurements, calculate the average coil radius R and its uncertainty. Check to see if the location of the tube is really at $R/2$.

From the current and geometry, calculate the magnetic field B at the center of the coil apparatus (including its uncertainty) from Eq. (6). Then, for each marked circle, calculate e/m with its uncertainty from Eq. (7) for each data point, and convince yourself that the units work out right. *Write a Perl program to help automate this task.*

Determine a value of e/m for each electron beam radius that best represents your data. Chances are that there is a systematic trend of e/m with circle radius, which, of course, is a systematic error.

Much of the reason for this error comes from the fact that the electrons feel the B -field before they exit through the hole in the top plate. They are already being deflected on their way out. The electron “gun” is located at 0.254 cm below the anode plate. As a first approximation, this can be compensated for by shifting the origin of the circular trajectory, so that it is located at a point below the plate. Geometry gives the correction as

$$\frac{e}{m} = \frac{2V}{B^2(r^2 + y_0^2)} , \quad (8)$$

where $y_0 = (2.54 \pm 0.01) \times 10^{-3}$ m. *Modify your Perl code to include this correction, and show the details of the geometry needed to obtain this correction in your Discussion section, thus providing a proof of Eq. (8).*

A second source of systematic error comes from the fact that the coils have physical extent — they are not exactly “line currents” located at a distance R from the coil axis. The actual magnetic field may be found by adding the contributions from small ring elements. From the Biot-Savart law, the field produced by a section of coil containing dN turns is

$$dB = \frac{\mu_0 I y^2 dN}{2[(x - x')^2 + y^2]^{3/2}} , \quad (9)$$

where x and y locate the coil element dN from the center of the coil, and x' is a point located on the central axis (see Fig. 4). If the “turn density” is constant, we may replace the infinitesimal element dN with the spatial element $n dx dy$, where $n = N / [w(R_b - R_a)]$, and integrate to find $B(x')$ from one of the coils. The result is, for *both* coils,

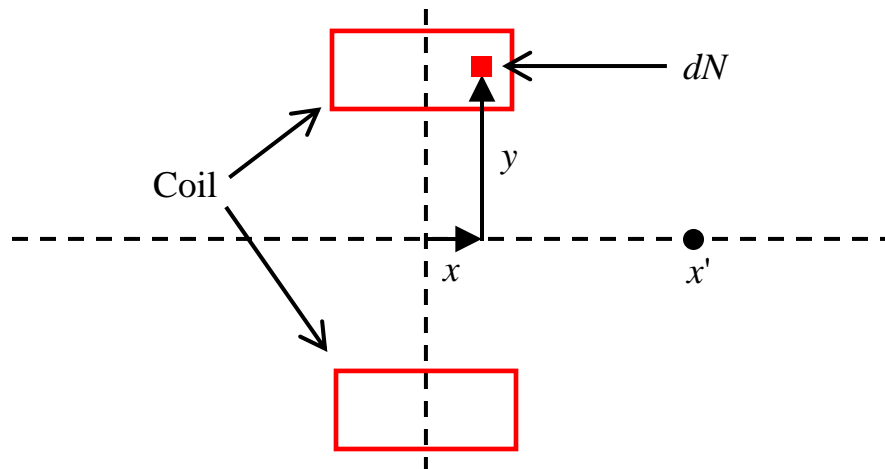


Figure 4. Geometry for calculating $B(x')$ from a coil with physical extent.

$$B(x') = \frac{8\mu_0 NI}{\sqrt{125}w(R_b - R_a)} \left[\left(\frac{w}{2} - x' \right) \ln \left(\frac{R_b + \sqrt{(w/2 - x')^2 + R_b^2}}{R_a + \sqrt{(w/2 - x')^2 + R_a^2}} \right) + \left(\frac{w}{2} + x' \right) \ln \left(\frac{R_b + \sqrt{(w/2 + x')^2 + R_b^2}}{R_a + \sqrt{(w/2 + x')^2 + R_a^2}} \right) \right] \quad (10)$$

Modify your Perl code to incorporate this systematic correction in the calculation of B , then recalculate e/m with this correction included, so that both systematic corrections are accounted for.

Plot all of your results of e/m versus electron beam radius (with and without the systematic corrections) on the same graph and discuss variations in the data (use *average* values of e/m for each electron beam radius). Your discussion should reflect the effect of including more *physics* into the analysis. Can you think of any other factor that could be influencing your results? For example, are relativistic corrections necessary for the electron velocity? Is the magnetic field *really* uniform, and how would a non-uniform field affect your calculation of e/m ? If effects such as these cause measurable effects, how may they have influenced your results? Include an estimate of the effect that the Earth's magnetic field would have on this experiment if not accounted for. (The Earth's magnetic field at Delaware, Ohio is given by the vector $\mathbf{B}_E = 19667\mathbf{i} - 2308\mathbf{j} + 49939\mathbf{k}$ nT, where the unit vectors denote the north, east, and downward vertical components, respectively.)

Finally, determine your *best* value for e/m and its uncertainty, reflecting all of your valid data and including the systematic corrections. Compare this result with the accepted value (see the Introduction section). Does your result agree within experimental uncertainty? If not, discuss possible sources of additional systematic error that may have crept in, giving realistic physical arguments why such sources have caused the value you measure to vary from the accepted value in the direction and magnitude that you see. Be specific and, if possible, quantitative in your arguments.