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The Betatron[1]

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THIS PAPER IS concerned with a new method for electron acceleration. The principles of the method, which was successfully accomplished for the first time at the University of Illinois (1, 2, 3), will be described briefly, since the type of accelerator used, the betatron, should find worthwhile applications in deep therapy. Some experimental results form the subject of a second paper (p. 120).

In the betatron energy is transferred to electrons by the accelerating effect of a time-varying magnetic field. Since a betatron is a powerful magnet, between the poles of which the electrons circulate in essentially one plane, the apparatus looks somewhat like a small cyclotron. It operates, however, with alternating current instead of direct current, and the process of acceleration is entirely different from that in the cyclotron. The betatron can accelerate particles whose velocity is very close to the velocity of light, such as electrons with energy in excess of half a million volts. Particles accelerated in a cyclotorn, on the other hand, must have a velocity much less than that of light, and therefore only heavy positive ions can be accelerated in it to appreciable energies.

Electrons from an electron gun or injector are shot into a circular path within a doughnut-shaped vacuum tube, while the magnetic field intensity is small. As these electrons circulate between the poles of the magnet, the magnetic field is increasing, and the time-rate of change of flux linking the orbit produces an energy gain per revolution equal to that produced by the voltage which would be read on a voltmeter connected to a one-turn coil placed at the orbit and recording instantaneous voltage.

Because of the great number of revolutions described by the electrons while the flux linkage is increasing, the energy in electron volts which is reached is roughly the same as the voltage generated in a secondary coil of the same number of turns placed around the magnetic core of the betatron and acting like a transformer secondary. Thus a betatron is similar to a transformer, but has the advantage that it is unnecessary to produce full voltage on a secondary coil and then apply that voltage to a high-vacuum x-ray tube. The electromotive force is instead continually applied directly to the electron stream.

Figure 1 shows the vacuum "doughnut" in which the electrons circulate many times, having traveled as far as 200 miles when they finally strike the injector, where they produce x-rays and scatter out of the doughnut into the room. The orbit-expanding coils are not energized until after the electrons have been accelerated; they disturb the flux distribution near the

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electron path, causing the electrons to spiral outward until they hit the first obstacle, the injector, which acts as the target. The injection time is indicated at A on the H curve in Fig. 1 ($H =$ the magnetic field), and the orbit is expanded to the target at the time indicated by C, when the energy is at a maximum. These processes are repeated in each cycle with a period of three-fourths of a cycle when no electrons are in the doughnut.

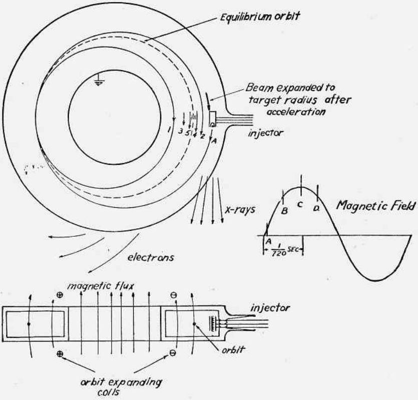

Fig. 1. The doughnut-shaped vacuum tube. Electrons are injected at time A in the magnetic cycle and directed against the target by the orbit expanding coils at time C.

The increase in flux linkage supplies momentum to the electron; and were the magnetic field at the orbit of the electron not increasing simultaneously, the orbit would become larger and larger and soon strike the outer wall of the vacuum chamber. To hold the electron orbit at a fixed radius it is necessary to increase the magnetic field, H, in proportion to the momentum, mv, produced by the increasing flux linkage. This requires a special distribution of magnetic field, as will be shown in the following analysis.

The radius of curvature r of the orbit of an electron of momentum mv is related to the magnetic field causing the circular motion by the equation

$$mv = (e/c)Hr \qquad (1)$$

in which e is the electronic charge in e.s.u., c is the speed of light in cm./sec., m is the mass of the electron in grams, v its velocity in cm./sec., and H the magnetic field in oersteds. If r is to be fixed, H must be proportional to mv. By Newton's second law the force, f, on an electron is the time-rate of change of momentum, $d(mv)/dt = f$, and this force is the energy gained per unit length of path.

For the present purposes, let us assume that the electron does travel in an orbit of fixed radius; then, since an induced voltage depends on the rate of change of flux linkage, the induced energy gain per centimeter, $f = (e/2\pi rc)d\phi/dt$, where $2\pi r$ is the circumference of the orbit and ϕ is the flux linking the orbit. Integrating

$$mv = \int_0^t f dt = (e/2\pi rc)(\phi - \phi_0) \qquad (2)$$

shows that the momentum is proportional to the change in flux linkage while the electron acquired momentum, mv. Combining (1) and (2) we get

$$\phi - \phi_0 = 2\pi r^2 H \qquad (3)$$

ϕ_0 is the flux linking the orbit when H is zero. This is a very fortunate result since, if both the field at the orbit and the flux within the orbit are produced in the same air gap in the magnetic circuit, ϕ_0 is zero and ϕ is proportional to H. The flux condition (3) is then automatically satisfied.

Although the above analysis has shown that, assuming r_0 is fixed, the flux condition (3) holds, the converse can and must be proved before the design problem can be considered as solved. This was done before the original betatron was constructed, and in the development of the complete theory (3) characteristics of the motion of electrons were discovered which play a vital part in the successful operation of the induction acceleration scheme. It must be possible to form a coherent beam capable of traveling a distance of the order of 100 miles in the vacuum tube without the electrons of the beam straying too far from the equilibrium orbit. A way must also be devised to introduce electrons so that they do not fly out of the acceleration region or hit the starting electrodes after a few revolutions. The theoretical treatment solved these problems.

The stray electrons which are deflected from the equilibrium orbit by scattering from residual gas molecules can be made to oscillate across the equilibrium orbit with a decreasing amplitude so that they eventually are brought back close to the orbit.

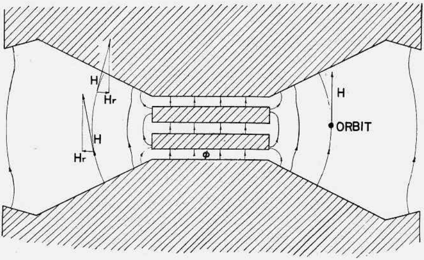

Fig. 2. Curvature of lines of force between the poles of the betatron. There is a radial component to the magnetic field everywhere except in the central plane. The radial component of magnetic field always forces electrons back toward this plane.

By shaping the magnetic field properly, the conditions for this oscillation can be fulfilled. For axial oscillation to occur, the magnetic lines of force must bulge outwardly between the poles, as shown in Figure 2. Then, if an electron deviates from the plane of the equilibrium orbit, it finds itself in a magnetic field with a slight radial component. This radial field is oppositely directed on opposite sides of the plane of the orbit, and it forces the electron, no matter in which direction it is displaced, back toward this plane. This bulging of the magnetic fields is easily accomplished by making the air gap between the poles increase with increasing radius. The pole face looks somewhat conical.

The condition imposed on the shape of the magnetic field for the production of radial oscillations is that the field should not decrease with radius faster than $1/r$. This can be understood by considering the forces required to hold the electron in a circular path and the forces supplied by the magnetic field. Figure 3 shows the force F_c necessary to hold an electron in a circle of radius r. This curve is hyperbolic, since $F = mv^2/r$ and v, the velocity of the electron, is changing so slowly that many focusing oscillations occur before it has altered much. The force, F_m, is the force supplied by the magnetic field. It is $F_m = (e/c)Hv$. If the field, H, is shaped so that F_m is less than F_c when r is less than the radius at the equilibrium orbit, then the magnetic force is not sufficient to hold the electron in at such a small radius. The electron thus will move toward the equilibrium orbit at r_0. If, at radii greater than r_0, F_m is greater than F_c, the magnetic force is more than that necessary to hold the electron in a circle, and the path curves in toward the equilibrium orbit. This shape of field is shown in Figure 3. Thus the stray electron will oscillate about r_0 and also across the plane of the orbit. Both of these oscillations are damped, that is, they have decreasing amplitudes. This is a result of the increasing strength of the magnetic field as time goes on; the amplitude is proportional to $1/H^{1/2}$. The effect of the increasing magnetic field is somewhat analogous to that of a stiffening spring which supports an oscillating mass.

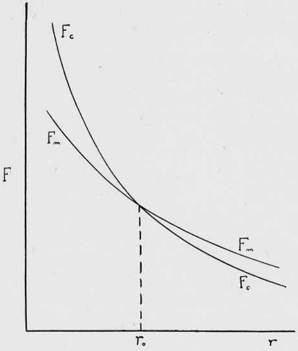

Fig. 3. F_c is the centripetal force mv^2/r required to hold an electron at radius r. $F_m = (e/c)Hr$ is the magnetic force which is supplied to the electron. Radial oscillation occurs about r_0.

It is because the oscillation of an electron is damped with a relatively large damping when H is small that electrons can be injected from a point at a radius slightly greater or smaller than r_0 and not hit the injector again on one of the first few revolutions. This makes possible the process which is used to get the electrons started.

In the original betatron this damping was made large by operating the magnet at a frequency high for such large flux densities in iron. Six hundred cycles per second alternating current were used, and it was estimated that the time average current striking the target was about 0.03 microamperes. In spite of the small magnitude of the current, the x-ray output in the beam was equivalent to that from about one gram of radium. At the energy which this betatron produced, 2.3 million volts, the x-rays tend to go forward in the direction which the electrons had when they struck the target. This simplifies protection of the operator.

The apparatus used to obtain the 20-million-volt results to be described in the following paper was constructed during a leave of absence from the University at the General Electric Company. It was an intermediate step toward construction following the university's 100-million-volt design. The 20-million-volt betatron (4), now in use at the University of Illinois, operates at a frequency of 180 cycles per second, and it is estimated that the average current reaching the target can be as high as one microampere. An output as great as 50 r/min. in a thick-walled ionization chamber at 70 cm. has been produced at the maximum energy. The x-rays go forward in a beam which is very pronounced. Although the output voltage of this betatron is less than ten times greater than the voltage of the small betatron, its weight, 3.5 tons, is about 35 times greater than the weight of the original.

While there was no thought that a 100-million-volt betatron could be useful in therapy, as a 20- to 30-million-volt model may prove to be, it was clear from the first operation of the original betatron, that we could plan to use this type of accelerator for certain cosmic ray work in our laboratory. In the design for the 100-million-volt betatron, that was intended for construction on the university campus, I have attempted to save space and materials by using proportions which are not the same as those chosen for the 20-million-volt betatron. There is a smaller fraction of the pole face used for focusing and a larger fraction used for acceleration or increasing flux linkage. While the 20-million-volt betatron gives 1 million volts per inch of pole diameter, the large accelerator is designed for 3/4 inch per million volts. Power and heating problems were also simplified by designing for a 60 cycles per second frequency, which results in a 24,000 kva. unit.

The name betatron, which is now in general usage among physicists, was chosen for the magnetic induction accelerator since it seems likely that the most useful applications of the betatron will involve the production of high-speed electrons or beta rays, as they are known in nuclear physics. When the Greek suffix, tron, is attached to the word beta, the name means the agency for producing high-energy electrons.

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